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# Cooperation in spare parts systems with penalty cost per unit backlogged

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# Research Background

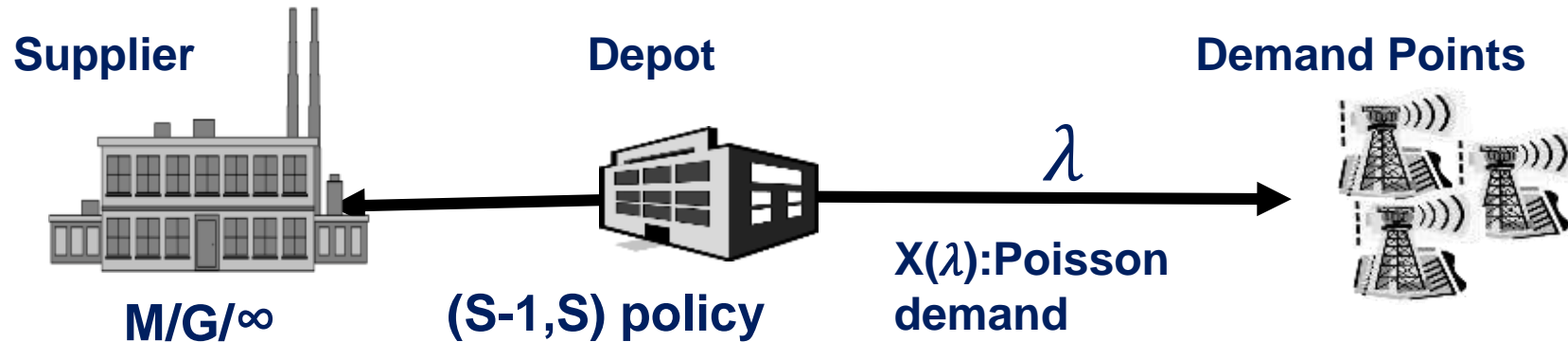
- Capital goods: durable goods that is used in the production of goods or services



## Common characteristics

- Expensive systems
- High downtime costs
- Expensive spare parts —————> High inventory cost
- Low utilization —————> great opportunity of cost reduction by inventory pooling

# A spare parts system with penalty cost per unit backlogged



## Relevant costs

$K(S)$ : Cost of inventory investment

$a$ : one time backlogging cost per unit

## Relevant performance measure

$BO(S, \lambda)$ : Average number of items backlogged per unit time

Due to PASTA property,  $BO(S, \lambda) = \lambda P(X(\lambda) \geq S)$

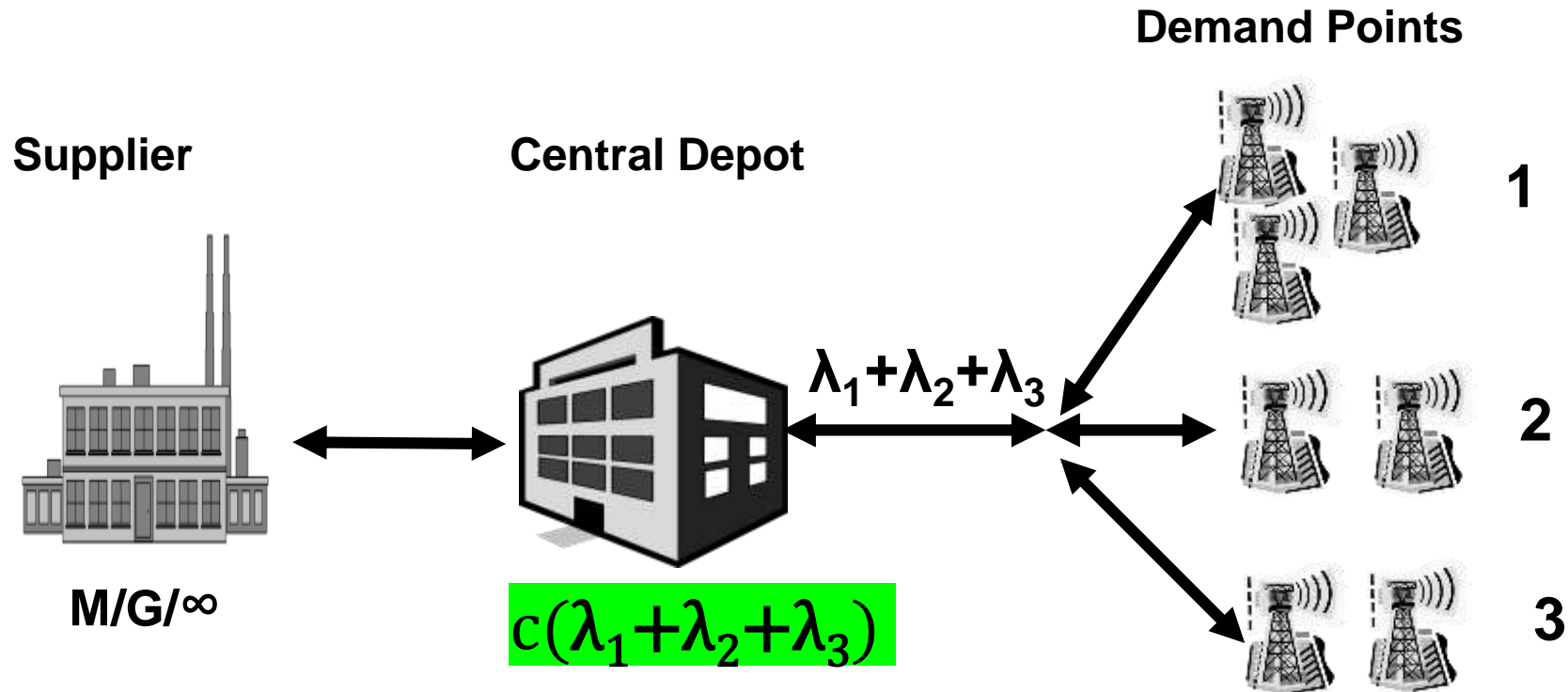
## Cost Function

$$C(S, \lambda) = K(S) + aBO(S, \lambda)$$

## Optimization Problem

$$c(\lambda) = \min_{S \in \mathbb{N}^0} C(S, \lambda)$$

# Cooperation Scenario



## Benefit:

Total cost of running a central depot will be lower.

## Question:

How to allocate the total cost among the coalition members?

## Cooperation under optimal inventory investment

- $R = \{1, \dots, r\}$  : the set of companies.
- Every company  $i \in R$  serves the demand with rate  $\lambda_i$ ,  
 $c(\lambda_i)$
- If coalition  $T \subseteq R$  is formed, demand rate is  $\lambda^T = \sum_{i \in T} \lambda_i$ ,  
 $c(\lambda^T)$
- *Single attribute game:*
  - *Attributes ( $\lambda$ ) are additive:  $\lambda^T = \sum_{i \in T} \lambda_i$*
  - *Coalition value is given by a single variable function:  $c(\lambda^T)$*

How to allocate the cost  $c(\lambda^R)$  among the members of the Grand coalition  $R$  ?

Is the core non-empty?



# Key Finding

## Theorem 1

- $y_i = \lambda_i c(\lambda^R) / \lambda^R$  for all  $i \in R$  is a PMAS and hence a core allocation.



## Academic Contribution

- New properties of performance measures (i.e., SH1)
- $C(S,\lambda)$  is not convex, subadditive or SH1 in general.
- Several subclasses of games with empty Cores
- (cooperation by inventory pooling and under service level constraint)
- Core allocations for the (sub) classes of games with nonempty Cores
- (cooperation under optimal inventory investment, etc..)